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$C_\lambda-$Rate Sequence Space Defined by a Modulus Function

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Abstract

Recall that a C_λ method is obtained by deleting a set of rows from the Cesáro matrix C_1. The purpose of this article is to introduce a new class of sequence space using a modulus function f, namely $C_\lambda-$rate sequence space. It is denoted by $C_\lambda(f, p, \pi)$, and study some inclusion relations and topological properties of this space.

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Introduction

The notion of modulus function was introduced by Nakano [11] and further investigated by Ruckle [12], Maddox [9], Tripathy and Chandra [15] and many others. A function $f : [0, \infty) \to [0, \infty)$ is called a modulus if
(i) $f(x) = 0$ if and only if $x = 0$,
(ii) $f(x + y) \le f(x) + f(y)$,
(iii) f is increasing,
(iv) f is continuous from the right at 0.

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It is immediate from(ii) and (iv) that f is continuous everywhere on $[0,\ \infty)$. It is easy to see that f_1+f_2 is a modulus function when f_1 and f_2 are modulus functions and that the function f^i (i is a positive integer), the composition of a modulus function f with itself i times is also a modulus function.

Let ℓ^0 be the space of all real sequences. For $1<p<\infty$, the Cesáro sequence space

$$ces_p=\left\{x\in\ell^0:\ \sum_{n=1}^{\infty}\left[\frac{1}{n}\sum_{k=1}^{n}|x_k|\right]^p<\infty\right\}$$

was first defined by Shiue in [14]. Various geometric properties of this space were studied by many others. The mentioned space was used in the theory of matrix operator and others. Also, it is used by Lee [1] and Lui, Wu, Lee [3]. The generalized Cesáro sequence space $ces\,(p)$ were introduced and studied by Sanhan and Suantai [13]. It is defined as follows

$$ces\,(p)=\left\{x\in\ell^0:\ \rho\,(\lambda x)<\infty\ \text{ for some }\lambda>0\right\},$$

where

$$\rho\,(x)=\sum_{n=1}^{\infty}\left[\frac{1}{n}\sum_{k=1}^{n}|x_k|\right]^{p_n}$$

is a convex modular on $ces\,(p)$. Bala, in [7] continued to work on Cesáro sequence space defined by a modulus function and to give some algebraic and topological properties.

Let w denote the space of all real or complex-valued sequence. It can be topologized with the seminorms $p_n(x)=|x_n|$, $(n=1,2,...)$, any vector subspace X of w is a sequence space. A sequence space X with a vector space topology τ, is a K-space provided that the inclusion map $i:(X,\tau)\to w$, $i\,(x)=x$, is continuous. If, in addition, τ is complete, metrizable and locally convex then (X,τ) is an FK-space. So an FK-space is a complete, metrizable locally convex topological vector space of sequences for which the coordinate functionals $P_n(x)=x_n$, $(n=1,2,...)$, are continuous. The basic properties of FK-spaces may be found in [18], [19] and [21].

Ruckle [12] used the idea of a modulus function f to construct a class of FK spaces

$$L(f)=\left\{x=(x_k):\sum_{k=1}^{\infty}f(|x_k|)<\infty\right\}.$$

Let $\pi=(\pi_n)$ be a sequence of positive numbers i.e, $\pi_n>0,\forall n\in\mathbb{N}$ and X an $FK-$space. We shall consider the sets of sequences $x=(x_n)$

$$X_\pi=\{x\in w:(\frac{x_n}{\pi_n})\in X\}.$$

The set X_π may be considered as FK-space.We shall call them as rate spaces (see, [4] and [5]).

Let F be an infinite subset of $\mathbb{N}$ and F as the range of a strictly increasing sequence of positive integers, say $F = \{\lambda(n)\}_{n=1}^{\infty}$. The Cesáro submethod C_λ is defined as

$$(C_\lambda x)_n = \frac{1}{\lambda(n)} \sum_{k=1}^{\lambda(n)} x_k, \ (n = 1, 2, ...),$$

where $\{x_k\}$ is a sequence of a real or complex numbers. Therefore, the C_λ-method yields a subsequence of the Cesáro method C_1, and hence it is regular for any λ. C_λ is obtained by deleting a set of rows from Cesáro matrix. The basic properties of C_λ-method can be found in [16] and [17]. We need the following inequality throughout the paper. Let $p = (p_k)$ be a sequence of positive real numbers with $G = \sup_k p_k$ and $D = \max(1, 2^{G-1})$. Then, it is well known that for all a_k, $b_k \in \mathbb{C}$, the field of complex numbers, for all $k \in \mathbb{N}$,

$$(1) \qquad |a_k + b_k|^{p_k} \leq D \ \left(|a_k|^{p_k} + |b_k|^{p_k}\right).$$

Also for any complex μ,

$$(2) \qquad \mu^{p_k} \leq \max(1, \mu^G)$$

see in [8].

Now we introduce the C_λ−rate sequence space $C_\lambda(f, p, \pi)$ using a modulus function f as follows

$$C_\lambda(f, p, \pi) = \left\{ x \in w : \sum_{n=1}^{\infty} \left[f\left(\frac{1}{\lambda(n)} \sum_{k=1}^{\lambda(n)} \left| \frac{x_k}{\pi_k} \right| \right) \right]^{p_n} < \infty \right\}.$$

Similarly, we can define that

$$C_\lambda(p, \pi) = \left\{ x \in w : \sum_{n=1}^{\infty} \left[\frac{1}{\lambda(n)} \sum_{k=1}^{\lambda(n)} \left| \frac{x_k}{\pi_k} \right| \right]^{p_n} < \infty \right\}.$$

MAIN RESULTS

THEOREM 1. *Let the sequence $p = (p_n)$ be bounded. Then the set $C_\lambda(f, p, \pi)$ is linear space over the complex field $\mathbb{C}$, for any modulus function f.*

Proof. Let $x, \ y \in C_\lambda(f, p, \pi)$. For $\alpha, \beta \in \mathbb{C}$, there exist integers M_α ve N_β such that $\alpha \leq M_\alpha$ and $\beta \leq N_\beta$. By definition of modulus function and inequalities (1) and (2) we can get

$$\begin{aligned}
&\sum_{n=1}^{\infty}\left[f\left(\frac{1}{\lambda(n)}\sum_{k=1}^{\lambda(n)}\left|\frac{\alpha\ x_k+\beta\ y_k}{\pi_k}\right|\right)\right]^{p_n}\\
\leq\ &\sum_{n=1}^{\infty}\left[f\left(\frac{1}{\lambda(n)}\sum_{k=1}^{\lambda(n)}\left|\frac{\alpha x_k}{\pi_k}\right|\right)+f\left(\frac{1}{\lambda(n)}\sum_{k=1}^{\lambda(n)}\left|\frac{\beta\ y_k}{\pi_k}\right|\right)\right]^{p_n}\\
\leq\ &D\left(\max(1,M_\alpha^G)\sum_{n=1}^{\infty}\left[f\left(\frac{1}{\lambda(n)}\sum_{k=1}^{\lambda(n)}\left|\frac{x_k}{\pi_k}\right|\right)\right]^{p_n}\right)\\
&+D\left(\max(1,N_\beta^G)\sum_{n=1}^{\infty}\left[f\left(\frac{1}{\lambda(n)}\sum_{k=1}^{\lambda(n)}\left|\frac{y_k}{\pi_k}\right|\right)\right]^{p_n}\right).
\end{aligned}$$

This implies that $\alpha x+\beta y\in C_\lambda(f,p,\pi)$, and completes the proof of Theorem.

THEOREM 2. *$C_\lambda(f,p,\pi)$ topological linear space paranormed by*

$$g^*(x)=\left(\sum_{n=1}^{\infty}\left[f\left(\frac{1}{\lambda(n)}\sum_{k=1}^{\lambda(n)}\left|\frac{x_k}{\pi_k}\right|\right)\right]^{p_n}\right)^{\frac{1}{M}},$$

where $G=sup_n p_n<\infty$ and $M=max\ (1,G)$.

The proof follows by using standart techniques and the fact that every paranormed space is a topological linear space $[20,\ p.\ 37]$. So we omit the details.

THEOREM 3. *$C_\lambda(f,p,\pi)$ is a Fréchet space paranormed by*

$$g^*(x)=\left(\sum_{n=1}^{\infty}\left[f\left(\frac{1}{\lambda(n)}\sum_{k=1}^{\lambda(n)}\left|\frac{x_k}{\pi_k}\right|\right)\right]^{p_n}\right)^{\frac{1}{M}}$$

where $G=sup_n p_n<\infty$ and $M=max(1,G)$.

Proof. In view of Theorem 2. it suffices to prove the completeness of $C_\lambda(f,p,\pi)$. $(x^{(i)})$ be any Cauchy sequence in $C_\lambda\ (f,p,\pi)$, where

$$\left(x^{(i)}\right)=(x_1^{(i)},\ x_2^{(i)},...),\ \forall i\in\mathbb{N}.$$

Then given any $\varepsilon>0$ there exist a positive integer N depending on ε such that

$g^*(x^{(i)}-x^{(j)})<\varepsilon,\ \forall i,j\geq N.$

Using the definition of paranorm we write

$$g^*(x^{(i)}-x^{(j)})=\left(\sum_{n=1}^{\infty}\left[f\left(\frac{1}{\lambda(n)}\sum_{k=1}^{\lambda(n)}\left|\frac{x_k^{(i)}-x_k^{(j)}}{\pi_k}\right|\right)\right]^{p_n}\right)^{\frac{1}{M}}<\varepsilon,\quad\forall i,j\geq N.$$

This implies that for each fixed k, $\left|\frac{x_k^{(i)}}{\pi_k} - \frac{x_k^{(j)}}{\pi_k}\right| \to 0$ as $i, j \to \infty$ and so $(x^{(i)})$ is any Cauchy sequence in $\mathbb{C}$, but $\mathbb{C}$ is complete so as $j \to \infty$ $x_k^{(j)} \to x_k$ $\forall k \in \mathbb{N}$. Now from (3) we have

$$\sum_{n=1}^{\infty} \left[f\left(\frac{1}{\lambda(n)} \sum_{k=1}^{\lambda(n)} \left| \frac{x_k^{(i)}}{\pi_k} - \frac{x_k^{(j)}}{\pi_k} \right| \right) \right]^{p_n} < \varepsilon^M, \text{ for each } i, j > N. \tag{4}$$

From (4), for any fixed natural number K, we have

$$\sum_{n=1}^{K} \left[f\left(\frac{1}{\lambda(n)} \sum_{k=1}^{\lambda(n)} \left| \frac{x_k^{(i)}}{\pi_k} - \frac{x_k^{(j)}}{\pi_k} \right| \right) \right]^{p_n} < \varepsilon^M, \quad \text{for each } i, j > N,$$

by taking $j \to \infty$ in the above expression we can get

$$\sum_{n=1}^{K} \left[f\left(\frac{1}{\lambda(n)} \sum_{k=1}^{\lambda(n)} \left| \frac{x_k^{(i)}}{\pi_k} - \frac{x_k}{\pi_k} \right| \right) \right]^{p_n} < \varepsilon^M, \text{ for each } \ i > N.$$

Since K is arbitrary by taking $K \to \infty$ we obtain

$$\sum_{n=1}^{\infty} \left[f\left(\frac{1}{\lambda(n)} \sum_{k=1}^{\lambda(n)} \frac{x_k^{(i)}}{\pi_k} - \frac{x_k}{\pi_k} \right) \right]^{p_n} < \varepsilon^M, \text{ for each } i > N,$$

that is $g^*(x^{(i)} - x) < \epsilon$ for each $i > N$.

To show that $x \in C_\lambda(f, p, \pi)$, let $j > N$ and fix n_0. Since $p_k/M \leq 1$ and $1 \leq M$ using Minkowski's inequality and definition of modulus function we obtain

$$\begin{aligned}
& \left(\sum_{n=1}^{n_0} \left[f\left(\frac{1}{\lambda(n)} \sum_{k=1}^{\lambda(n)} \left| \frac{x_k}{\pi_k} \right| \right) \right]^{p_n} \right)^{\frac{1}{M}} \\
= & \left(\sum_{n=1}^{n_0} \left[f\left(\frac{1}{\lambda(n)} \sum_{k=1}^{\lambda(n)} \left| \frac{x_k}{\pi_k} - \frac{x_k^{(j)}}{\pi_k} + \frac{x_k^{(j)}}{\pi_k} \right| \right) \right]^{p_n} \right)^{\frac{1}{M}} \\
\leq & \left(\sum_{n=1}^{n_0} \left[f\left(\frac{1}{\lambda(n)} \sum_{k=1}^{\lambda(n)} \left| \frac{x_k}{\pi_k} - \frac{x_k^{(j)}}{\pi_k} \right| \right) + f\left(\frac{1}{\lambda(n)} \sum_{k=1}^{\lambda(n)} \left| \frac{x_k^{(j)}}{\pi_k} \right| \right) \right]^{p_n} \right)^{\frac{1}{M}} \\
\leq & \left(\sum_{n=1}^{n_0} \left[f\left(\frac{1}{\lambda(n)} \sum_{k=1}^{\lambda(n)} \left| \frac{x_k}{\pi_k} - \frac{x_k^{(j)}}{\pi_k} \right| \right) \right]^{p_n} \right)^{\frac{1}{M}} + \left(\sum_{n=1}^{n_0} \left[f\left(\frac{1}{\lambda(n)} \sum_{k=1}^{\lambda(n)} \left| \frac{x_k^{(j)}}{\pi_k} \right| \right) \right]^{p_n} \right)^{\frac{1}{M}}. \\
\leq & \ \varepsilon + g^*(x^{(j)}).
\end{aligned}$$

This completes the proof.

The below theorem gives us the inclusion relations between $C_\lambda(f,p,\pi)$ and $C_\lambda(f,q,\pi)$ spaces.

THEOREM 4. *Let $p=(p_n)$ and $q=(q_n)$ are bounded sequences of positive real numbers with $0<p_n\le q_n<\infty$ for each n. Then for any modulus f, $C_\lambda(f,p,\pi)\subset C_\lambda(f,q,\pi)$.*

Proof. Let $x\in C_\lambda(f,p,\pi)$. Then we have

$$\sum_{n=1}^{\infty}\left[f\left(\frac{1}{\lambda(n)}\sum_{k=1}^{\lambda(n)}\left|\frac{x_k}{\pi_k}\right|\right)\right]^{p_n}<\infty.$$

Hence, since f non-decreasing, we get

$$f\left(\frac{1}{\lambda(n)}\sum_{k=1}^{\lambda(n)}\left|\frac{x_k}{\pi_k}\right|\right)\le 1$$

for sufficiently large n. Thus we have

$$\sum_{n=1}^{\infty}\left[f\left(\frac{1}{\lambda(n)}\sum_{k=1}^{\lambda(n)}\left|\frac{x_k}{\pi_k}\right|\right)\right]^{q_n}\le\sum_{n=1}^{\infty}\left[f\left(\frac{1}{\lambda(n)}\sum_{k=1}^{\lambda(n)}\left|\frac{x_k}{\pi_k}\right|\right)\right]^{p_n}<\infty.$$

This shows that $x\in C_\lambda(f,q,\pi)$ and completes the proof.

THEOREM 5. *If $r=(r_n)$ and $t=(t_n)$ are bounded sequences of positive real numbers with $0<r_n,t_n<\infty$ and $p_n=\min(r_n,t_n)$, then for any modulus f, $C_\lambda(f,q,\pi)=C_\lambda(f,r,\pi)\cap C_\lambda(f,t,\pi)$.*

Proof. Since $p_n=\min(r_n,t_n)$ we can write $p_n\le r_n$ and $p_n\le t_n$. It follows from Theorem 4 we obtain $C_\lambda(f,q,\pi)\subset C_\lambda(f,r,\pi)$ and $C_\lambda(f,q,\pi)\subset C_\lambda(f,t,\pi)$. For any complex μ, $\mu^{p_n}\le\max(\mu^{r_n},\mu^{t_n})$; thus $C_\lambda(f,r,\pi)\cap C_\lambda(f,t,\pi)\subset C_\lambda(f,q,\pi)$ and the proof is completed.

Consequently, we now give some information on multipliers for $C_\lambda(f,p,\pi)$. For any set E of sequences, the space of multipliers of E, denoted by $M(E)$, is given by

$$M(E)=\{a\in w:\ a.x\in E\text{ for all }x\in E\}.$$

THEOREM 6. *If $G=\sup_k p_k<\infty$, then for any modulus f, the inclusion $\ell_\infty\subset M(C_\lambda(f,p,\pi))$ is strict.*

Proof. Let $a=(a_k)\in\ell_\infty$. Then for $\forall k\in\mathbb{N}$ we write $a_k<1+[K]$ for some $K>0$, where $[K]$ denotes the integer part of K.. From (2), we obtain

$$\sum_{n=1}^{\infty}\left[f\left(\frac{1}{\lambda(n)}\sum_{k=1}^{\lambda(n)}\left|\frac{a_k\ x_k}{\pi_k}\right|\right)\right]^{p_n}\le(1+[K])^G\left(\sum_{n=1}^{\infty}\left[f\left(\frac{1}{\lambda(n)}\sum_{k=1}^{\lambda(n)}\left|\frac{x_k}{\pi_k}\right|\right)\right]^{p_n}\right).$$

Since $x\in C_\lambda(f,p,\pi)$ hence we get $\ell_\infty\subset M(C_\lambda(f,\ p,\ \pi))$.

We take modulus function f^u instead of f in the space $C_\lambda\ (f,p,\pi)$. Now we define the composite space $C_\lambda(f^u, p, \pi)$ as follows. For a fixed natural number u we define

$$C_\lambda(f,\ p,\ \pi) = \left\{ x \in w:\ \sum_{n=1}^{\infty} \left[f^u \left(\frac{1}{\lambda(n)} \sum_{k=1}^{\lambda(n)} \left| \frac{x_k}{\pi_k} \right| \right) \right]^{p_n} < \infty \right\}.$$

THEOREM 7. *Let f be a modulus function and $u \in \mathbb{N}$, then*

(*i*) *If $\lim_{t\to\infty} \frac{f(t)}{t} = \gamma > 0$ then $C_\lambda(f^u, p, \pi) \subset C_\lambda(p, \pi)$,*

(*ii*) *If there exists a positive constant δ such that $f(t) \leq \delta t$ at for all $t \geq 0$ then $C_\lambda(p, \pi) \subset C_\lambda(f^u, p, \pi)$.*

Proof. (*i*) Following the proof of Proposition of Maddox [6], we have $\gamma = \lim_{t\to\infty} \frac{f(t)}{t} = \inf\left\{ \frac{f(t)}{t} :\ t > 0 \right\}$. Let $\gamma > 0$. By definition of γ we have $\gamma t \leq f(t)$ for all $t > 0$. Since f is increasing we write $\gamma^2 t \leq f^2(t)$. So by induction we get $\gamma^u t \leq f^u(t)$. Let $x \in C_\lambda(f^u, p, \pi)$. Using inequality (2), we obtain

$$\begin{aligned} \sum_{n=1}^{\infty} \left(\frac{1}{\lambda(n)} \sum_{k=1}^{\lambda(n)} \left| \frac{x_k}{\pi_k} \right| \right)^{p_n} &\leq \sum_{n=1}^{\infty} \left[\gamma^{-u} f^u \left(\frac{1}{\lambda(n)} \sum_{k=1}^{\lambda(n)} \left| \frac{x_k}{\pi_k} \right| \right) \right]^{p_n} \\ &\leq \max(1, \gamma^{-uG}) \sum_{n=1}^{\infty} \left[f^u \left(\frac{1}{\lambda(n)} \sum_{k=1}^{\lambda(n)} \left| \frac{x_k}{\pi_k} \right| \right) \right]^{p_n} \end{aligned}$$

hence $x \in C_\lambda(p, \pi)$.

(*ii*) Since $f(t) \leq \delta t$ for all $t > 0$ and f is an increasing function we have

$$f^u(t) \leq \delta^u t$$

for each $\upsilon \in \mathbb{N}$. Let $x \in C_\lambda(p, \pi)$, then from inequality (2) we obtain

$$\begin{aligned} \sum_{n=1}^{\infty} \left[f^u \left(\frac{1}{\lambda(n)} \sum_{k=1}^{\lambda(n)} \left| \frac{x_k}{\pi_k} \right| \right) \right]^{p_n} &\leq \sum_{n=1}^{\infty} \left[\delta^u \left(\frac{1}{\lambda(n)} \sum_{k=1}^{\lambda(n)} \left| \frac{x_k}{\pi_k} \right| \right) \right]^{p_n} \\ &\leq \max(1, \delta^{uG}) \sum_{n=1}^{\infty} \left[\left(\frac{1}{\lambda(n)} \sum_{k=1}^{\lambda(n)} \left| \frac{x_k}{\pi_k} \right| \right) \right]^{p_n} < \infty \end{aligned}$$

and hence $x \in C_\lambda(f^u, p, \pi)$. This completes the proof.

THEOREM 8. *Let $m, u \in \mathbb{N}$ be such that $m \leq u$. If there exists a positive constant δ such that $f(t) \leq \delta t$ for all $t > 0$, then*

$$C_\lambda(p, \pi) \subseteq C_\lambda(f^m, p, \pi) \subseteq C_\lambda(f^u, p, \pi).$$

Proof. Let $r = u - m > 0$. Since $f(t) < \delta t$ we have

$$f^u(t) \leq M^r f^m(t) \leq M^u t,$$

where $M = 1 + [\delta]$. Let $x \in C_\lambda(p, \pi)$. By the above inequality, we obtain

$$\begin{aligned}\sum_{n=1}^{\infty}\left[f^u\left(\frac{1}{\lambda(n)}\sum_{k=1}^{\lambda(n)}\left|\frac{x_k}{\pi_k}\right|\right)\right]^{p_n} &\leq M^{rG}\sum_{n=1}^{\infty}\left[f^m\left(\frac{1}{\lambda(n)}\sum_{k=1}^{\lambda(n)}\left|\frac{x_k}{\pi_k}\right|\right)\right]^{p_n}\\ &\leq M^{vG}\sum_{n=1}^{\infty}\left(\frac{1}{\lambda(n)}\sum_{k=1}^{\lambda(n)}\left|\frac{x_k}{\pi_k}\right|\right)^{p_n}\end{aligned}$$

and hence the proof is completed.

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